INFLUENCE OF TEMPERATURE FIELD ON STABILITY OF FLUID MOTION BETWEEN ROTATING CYLINDERS

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The Rayleigh method is used to obtain relations giving the influence of the amount and direction of heat flow on the stability of a fluid between rotating cylinders.

One of the main peculiarities of fluid flows subjected to inertia force fields is the excitation of secondary flows which radically alter the nature of the flow, the heat transfer conditions, and the flow friction. Secondary flows result from nonuniformity of the body forces field in the system. The body force, referred to unit volume, may be expressed in terms of an inertial acceleration j and fluid density ρ:

$$F = j\rho.$$

Therefore, secondary flows may appear only in systems in which j or ρ change.

Depending on the nature of the change in body forces in a system, it may promote flow stabilization or destabilization. For example, in the flow of a fluid over a horizontal surface, the body forces due to the Earth's gravitational field stabilize the flow when the body force and its gradient have the same direction, and, conversely, have a destabilizing influence when these vector quantities have opposite directions.

Taylor [1] analyzed the stability of motion of a fluid between rotating cylinders under isothermal conditions by a perturbation method, and obtained good agreement between theory and experiment. An attempt was made in [2] to extend the small perturbation method to nonisothermal conditions. The authors limited attention to the case of a motionless outer cylinder, assumed a linear temperature distribution with radius, and represented the results of the theoretical investigation by the formula

$$\frac{\mathrm{Ta}}{\mathrm{Ta}_0} = \left(1 - \frac{1}{4}\beta \Delta t \,\mathrm{Pr}\right)^{-1}. \quad (1)$$

We shall analyze the stability of motion of a fluid between rotating cylinders under isothermal conditions by the Rayleigh method, described in [3].

In this method no account is taken of the influence of viscosity on the nature of the motion of randomly moving fluid particles, and therefore the moment of momentum of the particles is taken to be constant in this kind of motion.

If volume Δv of fluid with density ρ is displaced from radius r_0 to $r > r_0$, the centrifugal force acting on it will be

$$F_0 = M_0^2/\Delta v \rho_0 r^3, \quad (2)$$

and the force determining the pressure gradient and holding the rotating fluid in equilibrium at the new radius is

$$F = M^2/\Delta v \rho r^3. \quad (3)$$

Here $\rho\,(ur)^2$ is the moment of momentum.

The stability condition may therefore be written as

$$M^2/\rho - M_0^2/\rho_0 > 0$$

$$\rho\,(ur)^2 - \rho_0 (u_0 r_0)^2 > 0. \quad (4)$$

By expanding the function $\rho(ur)^2$ in a Taylor series in positive differences $r - r_0$, and limiting it to two terms of the expansion, we find that the stability condition may be written as

$$\frac{d\rho\,(ur)^2}{dr} > 0 \quad (5)$$

or,

$$\frac{1}{(ur)^2}\frac{d\,(ur)^2}{dr} + \frac{1}{\rho}\frac{d\rho}{dr} > 0. \quad (6)$$

It may be seen from this formula that for a positive density gradient (temperature decrease with radius), the nonisothermal feature increases the stability of the motion, while for a negative gradient it promotes flow destabilization.

We shall use (6) to analyze the stability of a fluid located between rotating cylinders in the presence of radial heat flow. In the absence of axial displacement and with fluid viscosity independent of temperature, the radial distribution of peripheral velocity is given by [4]

$$u = \omega_2 (Ar + r_2^2 B/r), \quad (7)$$

where

$$A = (\bar{r}_2^2 - \bar{\omega}_1)/(\bar{r}_2^2 - 1); \quad B = (\bar{\omega}_1 - 1)/(\bar{r}_2^2 - 1).$$

With the aid of (7), the first term of (6) is found to be

$$\frac{1}{(ur)^2}\frac{d\,(ur)^2}{dr} = \frac{1}{r_2}\,\frac{4\bar{r}}{\bar{r}^2 + C}. \quad (8)$$

Here

$$C = B/A = (\bar{\omega}_1 - 1)/(\bar{r}_2^2 - \bar{\omega}_1).$$

The density change in the system may be evaluated by means of the volumetric expansion β. For the positive and negative heat fluxes we shall assume the following dependences of $\rho = f(t)$:

$$\rho = \rho_2/[1 + \beta(t - t_2)], \quad (9)$$

$$\rho = \rho_1/[1 + \beta(t - t_1)]. \quad (10)$$

Confining our examination to small gaps between the cylinders, we shall use a linear relation between temperature and radius

$$t = a + br. \quad (11)$$

Formulas (9), (10), and (11) allow determination of the second term of (6). When $q > 0$ and $q < 0$ we obtain

$$\frac{1}{\rho}\frac{d\rho}{dr} = \frac{1}{r_2}\,\frac{K}{1 + K(1 - \bar{r})}; \quad (12)$$

$$\frac{1}{\rho}\frac{d\rho}{dr} = \frac{1}{r_2}\,\frac{K}{1 + K\left(\frac{1}{\bar{r}_2} - \bar{r}\right)}. \quad (13)$$

Here K is a dimensionless group characterizing the magnitude and direction of the heat load:

$$K = -\beta r_2 b = -\beta r_2 \frac{dt}{dr} = \frac{\beta}{\lambda}\, r_2 q. \quad (14)$$

Therefore, in accordance with (6), the condition for stable motion of the fluid takes the form:
when $q > 0$

$$\frac{4\bar{r}}{\bar{r}^2 + C} + \frac{K}{1 + K(1 - \bar{r})} > 0; \quad (15)$$

when $q < 0$

$$\frac{4\bar{r}}{\bar{r}^2 + C} + \frac{K}{1 + K(1/\bar{r}_2 - \bar{r})} > 0. \quad (16)$$

By equating (15) and (16) to zero, we find the critical relation for velocity $\bar{\omega}_1^*$, corresponding to the limit of stability:
when $q > 0$

$$\bar{\omega}_1^* = (R\bar{r}_2^2 - K)/(R - K), \quad (17)$$

when $q < 0$

$$\bar{\omega}_1^* = (S\bar{r}_2^2 - K)/(S - K). \quad (18)$$

Here

$$R = 4\bar{r}(1 + K) - 3\bar{r}^2 K, \quad S = 4\bar{r}(1 + K/\bar{r}_2) - 3\bar{r}^2 K.$$

Increase of parameter $\bar{\omega}_1$ is accompanied by increase of the gradient of the body force in the direction of the axis of rotation, and promotes flow destabilization. The motion will therefore be stable if $\bar{\omega}_1 < \bar{\omega}_1^*$.

For an isothermal flow regime (K = 0), formulas (17) and (18) reduce to the well-known relation between ratio of angular velocities and cylinder radii [3]

$$\bar{\omega}_{1_0}^* = \bar{r}_2^2. \quad (19)$$

It may be seen from (17) and (18) that the critical angular velocity ratio depends on the radius at which the fluid motion is examined (Fig. 1a). As is seen from the figure, positive heat flux, which creates negative density gradient, enlarges the region of stable regimes of motion, while negative heat flux causes this region to contract.

When the heat fluxes are small, loss of stable motion occurs simultaneously over the whole section of the gap. When the absolute values of heat flux are large, the liquid layers near the outer cylinder prove to be the most stable. Therefore, by putting $\bar{r} = 1$ in (17) and (18), we obtain the critical angular velocity ratio corresponding to loss of fluid motion stability in the entire gap (Fig. 1b).

The change of flow stability due to thermal action may be described by the ratio $\bar{\omega}_1^*/\bar{\omega}_{1_0}^*$. For $q > 0$ and $\bar{r} = 1$, we have, from (17) and (19)

$$\frac{\bar{\omega}_1^*}{\bar{\omega}_{1_0}^*} = 1 + \frac{1}{4} K \frac{\bar{r}_2^2 - 1}{\bar{r}_2^2}. \quad (20)$$

When comparing systems under isothermal and nonisothermal conditions, the critical regimes may be evaluated according to the value of angular velocity of the inner cylinder, for identical velocity of the outer. Then (20) takes the form

$$\frac{\omega_1^*}{\omega_{1_0}^*} = 1 + \frac{1}{4} K \frac{\bar{r}_2^2 - 1}{\bar{r}_2^2}. \quad (21)$$

In contrast to (1), this equation describes the influence of heat flux on stability of the fluid for any angular velocity, whose value must be taken into account when $\omega_{1_0}^*$ is defined.

Similary, for $q < 0$ and $\bar{r} = 1$, we obtain

$$\frac{\omega_1^*}{\omega_{1_0}^*} = \left[1 - \frac{1}{4} K \left(3 + \frac{1 - 4\bar{r}_2}{\bar{r}_2^2}\right)\right]\left(1 - K\,\frac{\bar{r}_2 - 1}{\bar{r}_2}\right)^{-1}. \quad (22)$$

Let us compare the results obtained with (1). For an outer cylinder at rest we have

$$\mathrm{Ta}/\mathrm{Ta}_0\,(\omega_1^*/\omega_{1_0}^*)^2. \quad (23)$$

For $\bar{r} = 1$, allowing for (21), we obtain

$$\frac{\mathrm{Ta}}{\mathrm{Ta}_0} = \left(1 + \frac{1}{4} K \frac{\bar{r}_2^2 - 1}{\bar{r}_2^2}\right). \quad (24)$$

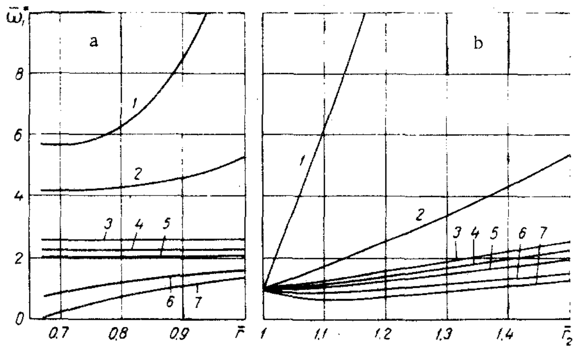

Fig. 1. a) Dependence of critical angular velocity ratio $\bar{\omega}_1^*$ or $\bar{r}$ when $\bar{r}_2$ = 1.5 and b) on $\bar{r}_2$ when $\bar{r}$ = = 1.0 for values of K equal to 100; 10; 1; 0; −1; −10; −100 (1–7, respectively).

For convenience of comparison of this expression with (1), we may put it in the form

$$\frac{\mathrm{Ta}}{\mathrm{Ta}_0} = \left(1 + \frac{1}{4}\,\beta\Delta t\,\frac{\bar{r}_2 + 1}{\bar{r}_2}\right)^2, \qquad (25)$$

where

$$\Delta t = t_1 - t_2.$$

There are, unfortunately, very few test data that could serve to check the relations obtained.

Becker and Kaye [5] have generalized the results of their tests and those of Bjerklund and Kaye on air heat transfer in the gap between cylinders, the inner one of which rotated while the outer was at rest. They found that under nonisothermal conditions, with a heat flux directed from the inner cylinder to the outer, $\mathrm{Ta}/\mathrm{Ta}_0$ = 1–1.17. Calculation of the ratio of critical Taylor numbers according to the data of [5] (Δt = 30°, r_2 = 1.24), with the aid of (1) and (25), gives 1.02 and 1.085, respectively. Therefore to check the theoretical relations we require experimental investigations at large heat loads and with various heat flux directions.

NOTATION

F—body force, referred to unit volume; j—inertial acceleration; $K = (\beta/\lambda)r_2 q$-dimensionless group; M—moment of momentum; P = $= f(r_1, r_2)$—dimensionless geometrical characteristic; Pr—Prandtl number; q—heat load; r_1, r_2—radii of inner and outer cylinders, respectively; r_m—mean gap radius; $\bar{r} = r/r_2$; $\bar{r}_2 = r_2/r_1$; Ta = $= \omega_1^2 r_m (r_2 - r_1)/\nu^2$ 1/P—critical value of modified Taylor number under nonisothermal conditions; Ta_0—the same under isothermal conditions; $\Delta t = t_1 - t_2$; t_1—temperature of inner cylinder surface; t_2—temperature of outer cylinder surface; u—circular velocity; ρ—fluid density; ω—angular velocity; ω_1—angular velocity of inner cylinder; ω_2—angular velocity of outer cylinder; $\omega_1 = \omega_1/\omega_2$; $\bar{\omega}_1^*$—critical ratio of angular velocities.

REFERENCES

1. G. J. Taylor, Phil. Transactions of the Roy. Soc. A, **223**, 1923.

2. K. M. Becker and J. Kaye, "Influence of a radial temperature gradient on instability of flow between two concentric cylinders, the inner rotating and the outer at rest," Trans. of the ASME, Ser. C, no. 2, 1962.

3. L. D. Landau and E. M. Lifshitz, Mechanics of Continuous Media [in Russian], Gostekhizdat, 1954.

4. S. M. Targ, Basic Problems in Laminar Flow Theory [in Russian], Gostekhizdat, 1951.

5. K. M. Becker and J. Kaye, "Study of a nonadiabatic flow in an annular channel with an inner rotating cylinder," Trans. of the ASME, Ser. C, no. 2, 1962.

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